

# Chapter 3

## Symbolic Keywords

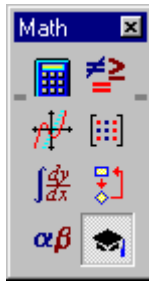
This chapter lists and describes Mathcad's symbolic keywords. The keywords are listed alphabetically.

### Accessing Symbolic Keywords

You can access symbolic keywords in two ways:

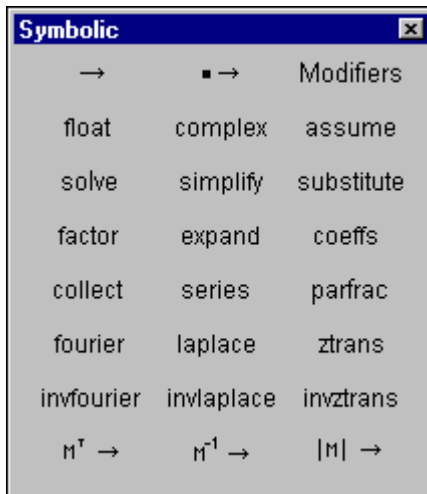
- Simply type in the keyword as shown for that keyword, or
- Select the keyword from the Symbolic toolbar.

1. First, click the **Symbolic Toolbar** button on the math menu:

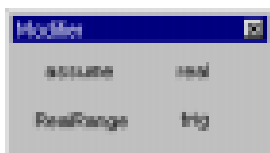


The Symbolic toolbar appears.

2. Click the button of the keyword that you want to use.



The **Modifiers** keyword button corresponds to symbolic modifiers.



The modifier **assume** is discussed on page 163. The other three modifiers, **real**, **RealRange** and **trig**, are used in some cases with the **simplify** keyword; refer to **simplify** on page 170 to find out how to use these modifiers.

Most of the keywords have equivalent menu choices on the **Symbolics** menu.



However, these menu choices are not “live,” which means they do not use any previous definitions in your worksheet and do not automatically update when you revise your worksheet.

## Finding More Information

Refer to the Resource Center QuickSheets for examples involving keywords. Select **Resource Center** from the **Help** menu. Then click on the QuickSheets icon and select a specific topic.

# Keywords

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## assume

Syntax `assume, constraint`

Description Imposes constraints on one or more variables according to the expression *constraint*. A typical constraint might be that  $var < 10$ .

`assume` can also constrain a variable to be real or to fall within a certain range of real values. Use the following modifiers:

`var=real` evaluates the expression on the assumption that the variable *var* is real;

`var=RealRange(a,b)` evaluates on the assumption that *var* is real and lies between *a* and *b*, where *a* and *b* are real numbers or infinity (type **[Ctrl][Shift]z** to display  $\infty$ ).

### Example

The screenshot displays four mathematical evaluations in Mathcad:

- Symbolic evaluation:**  $\int_0^{\infty} e^{-x^2} dx \rightarrow \frac{1}{2}\sqrt{\pi}$
- Complex evaluation:**  $e^{j\pi} \text{ complex} \rightarrow \cos(\pi - 0) + j \sin(\pi - 0)$
- Floating point evaluation:**  $\int_0^{\infty} e^{-x^2} dx \text{ float, 10} \rightarrow .8862269255$
- Constrained evaluation:**  $x \cdot \int_0^{\infty} x^{-\alpha+1} dx \text{ of assume, } \alpha > 1, \alpha = \text{real} \rightarrow \frac{x}{\alpha}$   
( $\alpha$  is constrained to be greater than 1 and real)

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## coeffs

Syntax `coeffs, var`

Description Finds coefficients of a polynomial when it is written in terms of ascending powers of the variable or subexpression *var*. Mathcad returns a vector containing the coefficients. The first element of the vector is the constant term and the last element is the coefficient of the highest order term in the polynomial.

See also `convert, parfrac` for example

Comments Another way to find the coefficients of a polynomial is to enclose the variable or subexpression *var* between the two editing lines and choose **Polynomial Coefficients** from the **Symbolics** menu.

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## collect

Syntax	collect, <i>var1</i> , <i>var2</i> , ... , <i>varn</i>
Description	Collects terms containing like powers of the variables or subexpressions <i>var1</i> through <i>varn</i> .
See also	expand for example
Comments	Another way to collect terms is to enclose the expression between the editing lines and choose <b>Collect</b> from the <b>Symbolics</b> menu

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## complex

Syntax	complex
Description	Carries out symbolic evaluation in the complex domain. Result is usually of the form $a + i \cdot b$ .
See also	assume for example
Comments	Another way to evaluate an expression in the complex domain is to enclose the expression between the editing lines and choose <b>Evaluate</b> ⇒ <b>Complex</b> from the <b>Symbolics</b> menu.

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## convert, parfrac

Syntax	convert, parfrac, <i>var</i>
Description	Converts an expression to a partial fraction expansion in the variable <i>var</i> .
Example	

Expanding expressions to partial fractions

$$\frac{2x^2 - 3x + 1}{x^3 + 2x^2 - 9x - 18} \text{ convert, parfrac, } x \rightarrow \frac{1}{3(x-3)} + \frac{14}{3(x+3)} - \frac{3}{(x+2)}$$

Use the "coeffs" keyword to treat an expression as a polynomial and write out the coefficients. Specify either a variable or a function as an argument to the keyword.

$$3 \cdot b \cdot x^4 - \pi \cdot x^2 + \frac{2}{3} \cdot x - .3 \cdot a \cdot b \text{ coeffs, } x \rightarrow \begin{bmatrix} -.3 \cdot a \cdot b \\ \frac{2}{3} \\ -\pi \\ 0 \\ 3 \cdot b \end{bmatrix}$$
$$\sin(x) + 2 \cdot \sin(x)^2 \text{ coeffs, } \sin(x) \rightarrow \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

**Comments** The symbolic processor tries to factor the denominator of the expression into linear or quadratic factors having integer coefficients. If it succeeds, it expands the expression into a sum of fractions with these factors as denominators. All constants in the selected expression must be integers or fractions; Mathcad does not expand an expression that contains decimal points.

Another way to convert an expression to a partial fraction is to click on the variable *var* anywhere in the expression. Then choose **Variable**⇒**Convert to Partial Fraction** from the **Symbolics** menu.

## expand

**Syntax** `expand, expr`

**Description** Expands all powers and products of sums in an expression except for the subexpression *expr*. The argument *expr* is optional. The entire expression is expanded if the argument *expr* is omitted.

**Example**

**Expanding expressions**

$$(x + y)^4 \text{ expand} \rightarrow x^4 + 4 \cdot x^3 \cdot y + 6 \cdot x^2 \cdot y^2 + 4 \cdot x \cdot y^3 + y^4$$

$$\cos(5 \cdot x) \text{ expand} \rightarrow 16 \cdot \cos(x)^5 - 20 \cdot \cos(x)^3 + 5 \cdot \cos(x)$$

$$(x + 1) \cdot (y + z) \text{ expand, } x + 1 \rightarrow (x + 1) \cdot y + (x + 1) \cdot z$$

**Factoring expressions**

$$8238913765711 \text{ factor} \rightarrow (73) \cdot (112861832407)$$

$$\frac{1}{x - 1} + \frac{x}{x + 3} - \frac{2 \cdot x}{x + 2} \text{ factor} \rightarrow \frac{-(2 \cdot x^2 - 9 \cdot x - 6 + x^3)}{[(x - 1) \cdot (x + 3) \cdot (x + 2)]}$$

$$x^2 - 2 \text{ factor, } \sqrt{2} \rightarrow (x + \sqrt{2}) \cdot (x - \sqrt{2})$$

**Collecting terms**

$$x^2 - a \cdot y \cdot x^2 + 2 \cdot y^2 \cdot x - x \text{ collect, } x \rightarrow (1 - a \cdot y) \cdot x^2 + (2 \cdot y^2 - 1) \cdot x$$

$$x^2 - a \cdot y \cdot x^2 + b \cdot y \cdot x - a \cdot x \cdot y \text{ collect, } x, y \rightarrow (1 - a \cdot y) \cdot x^2 + (b - a) \cdot y \cdot x$$

**Comments** Another way to expand an expression is to enclose the expression between the editing lines and choose **Expand** from the **Symbolics** menu.

## factor

**Syntax** `factor, expr`

**Description** Factors an expression into a product, if the entire expression can be written as a product. If the expression is a single integer, Mathcad factors it into powers of primes. If the expression is a polynomial or rational function, Mathcad factors it into powers of lower-order polynomials or rational functions. The argument *expr* is optional.

**See also** `expand` for example

**Comments** If you want to factor an expression over certain radicals, follow the `factor` keyword with a comma and the radicals.

You may be able to simplify an expression by factoring subexpressions, even if the expression taken as a whole can't be factored. To do so, enclose a subexpression between the editing lines and choose **Factor** from the **Symbolics** menu. You can also use the **Factor** menu command to factor an entire expression, but the **Symbolics** menu commands do not use any previous definitions in your worksheet and do not automatically update.

## float

Syntax	float, $m$
Description	Displays a floating point value with $m$ places of precision whenever possible. If the argument $m$ , an integer, is omitted, the default precision is 20.
See also	assume for example
Comments	Another way to perform floating point evaluation on an expression is to enclose the expression between the editing lines and choose <b>Evaluate</b> ⇒ <b>Floating Point</b> from the <b>Symbolics</b> menu. In the Floating Point dialog box, specify the number of digits to the right of the decimal point.

## fourier

Syntax	fourier, $var$
Description	Evaluates the Fourier transform of an expression with respect to the variable $var$ .

### Example

Dirac(t) fourier, t → 1      Press [Ctrl] [Shift] . to insert a transform keyword.

$$\frac{3}{1+x^2} \text{ invfourier, } x \rightarrow \frac{3}{2} \cdot \exp(-t) \cdot \Phi(t) + \frac{3}{2} \cdot \exp(t) \cdot \Phi(-t)$$

$$\exp(-a \cdot t) \text{ laplace, } t \rightarrow \frac{1}{(s+a)}$$

$$\frac{s}{s+a} \text{ invlaplace, } s \rightarrow -a \cdot \exp(-a \cdot t) + \text{Dirac}(t) \quad \leftarrow \text{Dirac}(t) \text{ is an impulse at } t=0. \text{ Although not numerically defined, Mathcad's symbolic processor recognizes this function.}$$

$$\sin\left(\frac{\pi}{2} \cdot t\right) \text{ ztrans, } t \rightarrow \frac{z}{(1+z^2)}$$

$$\frac{z}{z-2} \text{ invztrans, } z \rightarrow 2^n$$

*(See Appendix A, "Other Special Functions", for info on the Dirac delta function.)*

Comments      Mathcad returns a function of  $\omega$  given by:  $\int_{-\infty}^{+\infty} f(t)e^{-i\omega t} dt$  where  $f(t)$  is the expression to be transformed.

Mathcad returns a function in the variable  $\omega$  when you perform a Fourier transform because this is a commonly used variable name in this context. If the expression you are transforming already contains an  $\omega$ , Mathcad avoids ambiguity by returning a function of the variable  $\omega\omega$  instead.

Another way to evaluate the Fourier transform of an expression is to enter the expression and click on the transform variable. Then choose **Transform**⇒**Fourier** from the **Symbolics** menu.

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## invfourier

Syntax	invfourier, <i>var</i>
Description	Evaluates the inverse Fourier transform of an expression with respect to the variable <i>var</i> .
See also	fourier for example
Comments	<p>Mathcad returns a function of <math>t</math> given by: <math>\frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{i\omega t} d\omega</math> where <math>F(\omega)</math> is the expression to be transformed.</p> <p>Mathcad returns a function in the variable <math>t</math> when you perform an inverse Fourier transform because this is a commonly used variable name in this context. If the expression you are transforming already contains a <math>t</math>, Mathcad avoids ambiguity by returning a function of the variable <math>tt</math> instead.</p> <p>Another way to evaluate the inverse Fourier transform of an expression is to enter the expression and click on the transform variable. Then choose <b>Transform⇒Inverse Fourier</b> from the <b>Symbolics</b> menu.</p>

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## invlaplace

Syntax	invlaplace, <i>var</i>
Description	Evaluates the inverse Laplace transform of an expression with respect to the variable <i>var</i> .
See also	fourier for example
Comments	<p>Mathcad returns a function of <math>t</math> given by: <math>\frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} F(s) e^{st} dt</math> where <math>F(s)</math> is the expression to be transformed and all singularities of <math>F(s)</math> are to the left of the line <math>\text{Re}(s) = \sigma</math>.</p> <p>Mathcad returns a function in the variable <math>t</math> when you perform an inverse Laplace transform because this is a commonly used variable name in this context. If the expression you are transforming already contains a <math>t</math>, Mathcad avoids ambiguity by returning a function of the variable <math>tt</math> instead.</p> <p>Another way to evaluate the inverse Laplace transform of an expression is to enter the expression and click on the transform variable. Then choose <b>Transform⇒Inverse Laplace</b> from the <b>Symbolics</b> menu.</p>

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## invztrans

Syntax	invztrans, <i>var</i>
Description	Evaluates the inverse $z$ -transform of an expression with respect to the variable <i>var</i> .
See also	fourier for example
Comments	<p>Mathcad returns a function of <math>n</math> given by a contour integral around the origin: <math>\frac{1}{2\pi i} \int_C F(z) z^{n-1} dz</math></p> <p>where <math>F(z)</math> is the expression to be transformed and <math>C</math> is a contour enclosing all singularities of the integrand.</p> <p>Mathcad returns a function in the variable <math>n</math> when you perform an inverse <math>z</math>-transform since this is a commonly used variable name in this context. If the expression you are transforming already contains an <math>n</math>, Mathcad avoids ambiguity by returning a function of the variable <math>nn</math> instead.</p> <p>Another way to evaluate the inverse <math>z</math>-transform of an expression is to enter the expression and click on the transform variable. Then choose <b>Transform</b> <math>\Rightarrow</math> <b>Inverse Z</b> from the <b>Symbolics</b> menu.</p>

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## laplace

Syntax	laplace, <i>var</i>
Description	Evaluates the Laplace transform of an expression with respect to the variable <i>var</i> .
See also	fourier for example
Comments	<p>Mathcad returns a function of <math>s</math> given by: <math>\int_0^{+\infty} f(t) e^{-st} dt</math>, where <math>f(t)</math> is the expression to be transformed.</p> <p>Mathcad returns a function in the variable <math>s</math> when you perform a Laplace transform since this is a commonly used variable name in this context. If the expression you are transforming already contains an <math>s</math>, Mathcad avoids ambiguity by returning a function of the variable <math>ss</math> instead.</p> <p>Another way to evaluate the Laplace transform of an expression is to enter the expression and click on the transform variable. Then choose <b>Transform</b> <math>\Rightarrow</math> <b>Laplace</b> from the <b>Symbolics</b> menu.</p>



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## series

Syntax series, var=z, m

Description Expands an expression in one or more variables, var, around the point z. The order of expansion is m. Arguments z and m are optional. By default, the expansion is taken around zero and is a polynomial of order six.

### Example

Generating a series around the point x=0:

$$\ln(x + y) \text{ series, } x \rightarrow \ln(y) + \frac{x}{y} - \frac{1}{2} \frac{x^2}{y^2} + \frac{1}{3} \frac{x^3}{y^3} - \frac{1}{4} \frac{x^4}{y^4} + \frac{1}{5} \frac{x^5}{y^5}$$

Generating a series for sin(x) with order 6:

$$\sin(x) \text{ series, } x, 6 \rightarrow x - \frac{1}{6} x^3 + \frac{1}{120} x^5$$

Generating a series around the point x=1 and y=0 but show only those terms whose exponents sum to less than 3:

$$e^x + y \text{ series, } x=1, y, 3 \rightarrow \exp(1) + \exp(1) \cdot (x - 1) + y + \frac{1}{2} \exp(1) \cdot (x - 1)^2$$

↑  
Press [Ctrl] = for the equal sign .

### Comments

Mathcad finds Taylor series (series in nonnegative powers of the variable) for functions that are analytic at 0, and Laurent series for functions that have a pole of finite order at 0. To develop a series with a center other than 0, the argument to the **series** keyword should be of the form var=z, where z is any real or complex number. For example, **series, x=1** expands around the point x=1. Press [Ctrl] = for the equal sign.

To expand a series around more than one variable, separate the variables by commas. The last line in the example above shows an expression expanded around x and y.

Another way to generate a series expansion is to enter the expression and click on a variable for which you want to find a series expansion. Then choose **Variable⇒Expand to Series** from the **Symbolics** menu. A dialog box will prompt you for the order of the series. This command is limited to a series in a single variable; any other variables in the expression will be treated as constants. The results also contain the error term using the O notation. Before you use the series for further calculations, you will need to delete this error term.

When using the approximations you get from the symbolic processor, keep in mind that the Taylor series for a function may converge only in some small interval around the center. Furthermore, functions like **sin** or **exp** have series with infinitely many terms, while the polynomials returned by Mathcad have only a few terms (how many depends on the order you select). Thus, when you approximate a function by the polynomial returned by Mathcad, the approximation will be reasonably accurate close to the center, but may be quite inaccurate for values far from the center.

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## simplify

Syntax      simplify

Description      Simplifies an expression by performing arithmetic, canceling common factors, and using basic trigonometric and inverse function identities.

To control the simplification, use the following modifiers:

`assume=real` simplifies on the assumption that all the indeterminates in the expression are real;

`assume=RealRange(a,b)` simplifies on the assumption that all the indeterminates are real and are between  $a$  and  $b$ , where  $a$  and  $b$  are real numbers or infinity (**[Ctrl]Z**);

`trig`, simplifies a trigonometric expression by applying only the following identities:

$$\sin(x)^2 + \cos(x)^2 = 1 \quad \cosh(x)^2 - \sinh(x)^2 = 1,$$

but does not simplify the expression by simplifying logarithms, powers, or radicals.

### Example

$\frac{x^2 - 3x - 4}{x - 4} + 2x - 5 \text{ simplify} \rightarrow 3x - 4$
$e^{2 \ln(a)} \text{ simplify} \rightarrow a^2$
$\sin(\ln(a \cdot b))^2 \text{ simplify} \rightarrow 1 - \cos(\ln(a) + \ln(b))^2$
$\sin(\ln(a \cdot b))^2 \text{ simplify, trig} \rightarrow 1 - \cos(\ln(a \cdot b))^2$
$(2^b)^c \text{ simplify} \rightarrow (2^b)^c$
$(2^b)^c \text{ simplify, assume = real} \rightarrow 2^{b \cdot c} \quad \leftarrow \text{ Press [Ctrl] = for the equal sign.}$
$\sqrt{x^2} \text{ simplify} \rightarrow \text{csgn}(x) \cdot x$
$\sqrt{x^2} \text{ simplify, assume = RealRange}(-10, -5) \rightarrow -x \quad \leftarrow \text{ Press [Ctrl] = for the equal sign.}$

### Comments

You can also simplify an expression by placing it between the two editing lines and choosing **Simplify** from the **Symbolics** menu. This method is useful when you want to simplify parts of an expression. Mathcad may sometimes be able to simplify parts of an expression even when it cannot simplify the entire expression. If simplifying the entire expression doesn't give the answer you want, try selecting subexpressions and choosing **Simplify** from the **Symbolics** menu. If Mathcad can't simplify an expression any further, you'll just get the original expression back as the answer.

In general, when you simplify an expression, the simplified result will have the same numerical behavior as the original expression. However, when the expression includes functions with more than one branch, such as square root or the inverse trigonometric functions, the symbolic answer may differ from a numerical answer. For example, simplifying  $\text{asin}(\sin(\theta))$  yields  $\theta$ , but this equation holds true numerically in Mathcad only when  $\theta$  is a number between  $-\pi/2$  and  $\pi/2$ .

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## solve

Syntax solve, var

Description Solves an equation for the variable *var* or solves a system of equations for the variables in a vector *var*.

### Examples

$$A1 = \frac{L}{r^2} + 2 \cdot C \text{ solve, } r \rightarrow \begin{bmatrix} \frac{1}{(A1 - 2 \cdot C)} \cdot \sqrt{(A1 - 2 \cdot C) \cdot L} \\ -1 \\ \frac{-1}{(A1 - 2 \cdot C)} \cdot \sqrt{(A1 - 2 \cdot C) \cdot L} \end{bmatrix}$$

a := 34

$$\frac{1}{2} \cdot x^2 + x = -2 + a \text{ solve, } x \rightarrow \begin{bmatrix} -1 + \sqrt{65} \\ -1 - \sqrt{65} \end{bmatrix} \text{ Use [Ctrl] = for the equal sign.}$$
$$\frac{\alpha \cdot f + 1}{f - \beta} = e^{-\alpha} \text{ solve, } f \rightarrow \frac{-(1 + \exp(-\alpha) \cdot \beta)}{(\alpha - \exp(-\alpha))}$$
$$x^3 - 5 \cdot x^2 - 4 \cdot x + 20 > 0 \text{ solve, } x \rightarrow \begin{bmatrix} (-2 < x) \cdot (x < 2) \\ 5 < x \end{bmatrix}$$

$e^t + 1$  solve, t → i · π You don't need =0 when finding roots.

Example 1: Solving equations, solving inequalities, and finding roots.

Using the "solve" keyword (press [Ctrl]+[Shift]+Period):

$$\begin{bmatrix} x + 2 \cdot \pi \cdot y = a \\ 4 \cdot x + y = b \end{bmatrix} \text{ solve, } \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{(-1 + 8 \cdot \pi)} \cdot (2 \cdot \pi \cdot b - a) \\ \frac{-(-4 \cdot a + b)}{(-1 + 8 \cdot \pi)} \end{bmatrix}$$

Using a solve block:

Given  $x + 2 \cdot \pi \cdot y = a$  <- Use [Ctrl]= to type the equal sign.  
 $4 \cdot x + y = b$

Find(x, y) →  $\begin{bmatrix} \frac{1}{(-1 + 8 \cdot \pi)} \cdot (2 \cdot \pi \cdot b - a) \\ \frac{-(-4 \cdot a + b)}{(-1 + 8 \cdot \pi)} \end{bmatrix}$

Example 2: Solving a system of equations symbolically.

### Comments

Solving equations symbolically is far more difficult than solving them numerically. The symbolic solver sometimes does not give a solution. Many problems can only be solved via numerical approach and many more yield symbolic solutions too lengthy to be useful.

Another way to solve for a variable is to enter the equation, click on the variable you want to solve for in an equation, and choose **Variable**⇒**Solve** from the **Symbolics** menu.

You can use either the symbolic **solve** keyword or a solve block, as illustrated above, to solve a system of equations symbolically. No initial guess values are necessary for symbolic schemes

## substitute

Syntax      substitute,  $var1 = var2$

Description      Replaces all occurrences of a variable  $var1$  with an expression or variable  $var2$ . Press [Ctrl] = for the equal sign.

Example

To substitute  $x$  for  $z$  in the expression below, use the "substitute" keyword and an argument indicating which variable to replace with which expression. Use [Ctrl] = for the equal sign in the argument.

$$z^2 + \frac{2}{z} \text{ substitute, } z = x \rightarrow x^2 + \frac{2}{x}$$

Substituting  $f(\sin(x))$  for  $y$ :

$$\sqrt{1 + y^2} \text{ substitute, } y = f(\sin(x)) \rightarrow \sqrt{1 + f(\sin(x))^2}$$

Comments      Mathcad does not substitute a variable for an entire vector or a matrix. You can, however, substitute a scalar expression for a variable that occurs in a matrix. To do so, follow these steps:

1. Select the expression that will replace the variable and choose **Copy** from the **Edit** menu.
2. Click on an occurrence of the variable you want to replace and choose **Variable**⇒**Substitute** from the **Symbolics** menu. You can also use this menu command to perform a substitution in any expression.

## ztrans

Syntax      ztrans,  $var$

Description      Evaluates the  $z$ -transform of an expression with respect to the variable  $var$ .

See also      fourier for example

Comments      Mathcad returns a function of  $z$  given by:  $\sum_{n=0}^{+\infty} f(n)z^{-n}$ , where  $f(n)$  is the expression to be transformed.

Mathcad returns a function in the variable  $z$  when you perform a  $z$ -transform since this is a commonly used variable name in this context. If the expression you are transforming already contains a  $z$ , Mathcad avoids ambiguity by returning a function of the variable  $zz$  instead.

Another way to evaluate the  $z$ -transform of an expression is to enter the expression and click on the transform variable. Then choose **Transform**⇒**Z** from the **Symbolics** menu.